the boundary conditions, are obtained from measurements of the end deflections and rotations of the bar caused by a lateral load.

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Axisymmetric Dynamic Buckling of Clamped Shallow Spherical and Conical Shells under Step Loads

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THE purpose of this short Note is to present some additional numerical results of the axisymmetric dynamic buckling of clamped shallow spherical shells under step loads of infinite duration and to present, for the first time, similar numerical results for clamped shallow conical shells under the same type of loads.

It is shown that when the critical axisymmetric dynamic buckling loads for the clamped shallow spherical shell are obtained for certain discrete values of the geometric parameter λ, for which this information had not been obtained previously, the axisymmetric dynamic buckling curve (p_D vs λ) is modified in such a way that a striking similarity between the axisymmetric dynamic and the axisymmetric static (p_s vs λ) buckling curves becomes apparent. Indeed, it is shown that a two-dimensional translation of the axisymmetric dynamic buckling curve brings it into good agreement with the axisymmetric static buckling curve for a significant range of the geometric parameter λ.

Similar, and even more striking, results are observed between the axisymmetric dynamic and static buckling curves for the clamped conical shell under the same type of load.

The system of differential equations that govern the nonlinear axisymmetric oscillations of clamped shallow spherical and conical shells under step loads of infinite duration can be written in the nondimensional forms

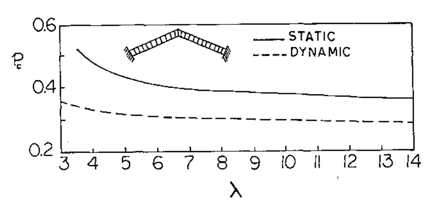

Fig. 1 Axisymmetric static and dynamic buckling behavior of shallow clamped conical caps under uniform pressure.

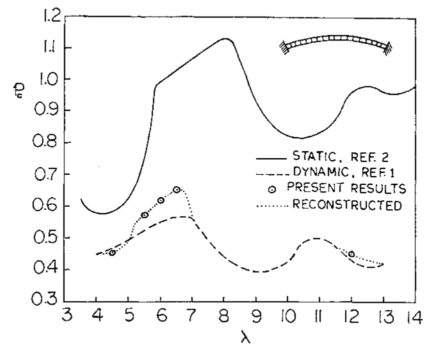

Fig. 2 Axisymmetric static and dynamic buckling behavior of shallow clamped spherical caps under uniform pressure.

$$\nabla^4 w = g_1 + (1/x)\,(\Phi w')' + 4p - \ddot{w} \tag{1}$$

$$(x\Phi')' - (1/x)\Phi + g_2 = -\tfrac{1}{2}(w')^2 \tag{2}$$

where

$$g_1 = \begin{cases} (1/x)(x\Phi)' & \text{for the spherical shell} \\ (\lambda/2x)\,\Phi' & \text{for the conical shell} \end{cases}$$
$$g_2 = \begin{cases} xw' & \text{for the spherical shell} \\ (\lambda/2)w' & \text{for the conical shell} \end{cases} \tag{3}$$

and

$$\nabla^2 \equiv (\ \ )'' + (1/x)(\ \ )' \tag{4}$$

Primes and dots over quantities signify differentiation with respect to the nondimensional base-plane radius x and the nondimensional time τ. Finally w and Φ are nondimensional transverse displacement and stress functions, respectively, and p is the nondimensional load parameter.

The boundary and initial conditions for both the clamped shallow spherical and conical caps under step loads of infinite duration are the same as those given in Ref. 1 for the clamped shallow spherical shell. Moreover, the numerical procedure described in Ref. 1 is used in the present study. In order to begin the numerical computations for the clamped shallow conical shell, the function $g_1(x = 0)$ is assumed to be the same as $g_1(x = 0)$ for the clamped shallow spherical shell. This is consistent with the assumption that a spherical inclusion exists in the vicinity of the apex of the conical cap.[4] Axisymmetric static buckling behavior of the clamped shallow conical shell is analyzed also by discarding the inertia term in Eq. (1).

The axisymmetric static and dynamic buckling curves for the clamped shallow conical shell are shown in Fig. 1 and the corresponding numerical data from which the curves were

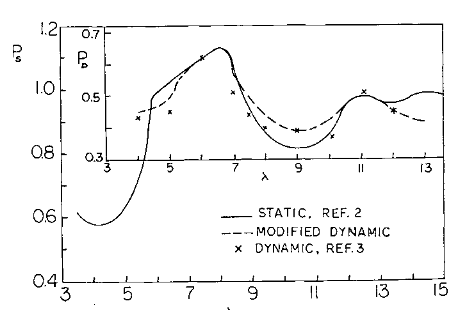

Fig. 3 Comparison of axisymmetric static and dynamic buckling curves of shallow spherical caps under uniform pressure.

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Table 1 Axisymmetric static and dynamic buckling loads of shallow conical caps

λ	3	4	5	6	7	8	9	10	11	12	13	14
p_s	...	0.470	0.430	0.405	0.400	0.388	0.380	0.374	0.370	0.367	0.365	0.363
p_D	0.35 0.36	0.33	0.32	0.31	0.30 0.31	0.30 0.31	0.30	0.29 0.30	0.29	0.29	0.29	0.29

Table 2 Axisymmetric static and dynamic buckling loads of shallow spherical caps

λ	4	4.5	5	5.5	6	6.5	7	7.5	8	9	10	11	12	13	14	15
p_s Ref. 2	0.578	...	0.629	0.762 0.763	0.995	...	1.068	...	1.130	0.931 0.940	0.822 0.830	0.83 0.84	0.96 0.97	0.96 0.97	0.98 0.99	0.98 0.99
p_D Ref. 1	0.45	...	0.49	...	...	...	0.56	0.50	0.44	0.39	0.42	0.50	...	0.42	...	...
p_D present results	...	0.46	...	0.57	0.62	0.65	...	...	...	...	...	...	0.45	...	...	...

constructed are given in Table 1. In Table 1, p_s and p_D refer to the axisymmetric static and dynamic buckling loads, respectively.

The axisymmetric static and dynamic buckling curves for the clamped shallow spherical shell are shown in Fig. 2 and the corresponding numerical data are given in Table 2. The dashed curve of Fig. 2 is the axisymmetric dynamic buckling curve obtained by plotting the results obtained from Ref. 1, the numerical data of which is shown in Table 2. The dotted curve is a modification of the dashed curve arrived at by calculating the dynamic buckling loads for values of λ intermediate to those considered in Ref. 1 and using the same time response.

The striking similarity between the axisymmetric static and dynamic buckling curves for both the clamped shallow spherical and conical shells is apparent from an inspection of Figs. 1 and 2; the similarity being more pronounced for the conical shell. A very satisfactory empirical relation between the axisymmetric static and dynamic buckling loads for the clamped shallow conical shell for the range of λ considered in the present study is

$$(p_D)_\lambda = (p_s)_{\lambda+1.9} - 0.07 \tag{5}$$

Equation (5) implies that, if the axisymmetric dynamic buckling curve for the conical shell is translated 1.9 λ units to the right and 0.7 p units upward, then it will coincide closely with the corresponding axisymmetric static buckling curve. The maximum relative error between values of the dynamic buckling loads obtained from Eq. (5) and those shown in Table 1 is approximately 2%.

The similarity between the axisymmetric static and dynamic buckling curves for clamped shallow spherical shells is not as pronounced for the full range of λ considered, however, striking similarities exist for a significant range of λ. Figure 3 shows the axisymmetric static and dynamic buckling curves for the clamped shallow spherical shell where the dynamic buckling curve has been translated 1.4 λ units to the right and 4.8 p_s units upward. Inspection of these curves shows the similarity for the range $5.5 < \lambda < 11.5$ of the upper abscissa. The cross marks on the figure indicate the critical dynamic buckling loads obtained by Stephens and Fulton[3] using a larger response time. The reduction of the dynamic buckling loads, especially for λ = 7.5, 8, and 10 (upper abscissa), increases the similarity of the curves.

The conclusions suggested by this Note are: that there is a striking similarity between the axisymmetric static and dynamic buckling curves for both the clamped shallow spherical and conical shells; and if there exists a relationship between the axisymmetric static and dynamic buckling curves for these shells it must depend on λ. The fact that the uniform pressure of infinite duration shifts the dynamic buckling curve not only downward, as is expected because of the inertia force, but also to the left is noteworthy; the reasons for which are worth exploring.

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Townsend Structure Parameters

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Nomenclature

K_1,K_2	=	Townsend's turbulence parameters [Eqs. (1) and (2)]
u,v,w	=	instantaneous values of velocity fluctuations in x,y, and z directions, respectively
x,y,z	=	distances in the axial, radial, and circumferential directions, respectively
$\langle\ \rangle$	=	time average
$\langle q^2\rangle$	=	$\langle u^2\rangle + \langle v^2\rangle + \langle w^2\rangle$
Re	=	Reynolds number based on diameter of pipe and mean velocity
δ^*	=	boundary-layer displacement thickness
θ	=	boundary-layer momentum thickness
δ^{**}	=	boundary-layer energy thickness

TOWNSEND[1] had defined two parameters; namely

$$K_1 = [\langle v^2\rangle - \langle w^2\rangle]/[\langle v^2\rangle + \langle w^2\rangle] \tag{1}$$

and

$$K_2 = [3\ \langle u^2\rangle/\langle q^2\rangle] - 1 \tag{2}$$

These parameters indicate the relative values of three intensities. Townsend[2] has also used these parameters to predict the equilibrium structure in fully strained turbulence. The author has adopted these parameters to investigate the structure of the diverging flow in a conical diffuser. The conical diffuser has an 8° solid angle, an area ratio of 4:1, and an inlet diameter of 10 cm. A pipe, 166 cm in length and

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